DRiFt model a possible link between regionalization and hydrological simulation

F. GIANNONI, G. ROTH & R. RUDARI

CIMA- Università di Genova Via Cadorna 7, 17100 Genova,
e-mail: gianna@diam.unige.it, *giorgio@diam.unige.it, roby@diam.unige.it*

ABSTRACT

Under the assumption of homogeneity, a regional dimensionless growth curve can be used to study the discharge variable distribution performing a regional analysis also in non-monitored basins. The determination of the local discharge index value, which makes dimensional the growth curve in each site, is a critical issue: in gauged sites the time series average sometimes is sufficient, while in non-gauged sites one is usually forced to perform a regression for the estimation of the index value.

In this work, this problem is solved by coupling rainfall regional statistics with DRiFt, a semi-distributed rainfall-runoff model based on a morphologic approach. The model is able to consider the topography of each site analyzed and the spatial variability of soil characteristics and rainfall patterns, allowing us to take into account the great site sensitivity of the discharge variable. The utilization of this hydrologic model in a regionalization procedure was achievable because of the invariance of the calibrated parameters within the homogeneous region considered. This model solves the tricky problem of finding a correlation among the discharge index value and some physical characteristics of non-gauged sites and it is proved to correctly reproduce the statistics of gauged ones. Moreover, using the rainfall time series information, the uncertainty in the discharge index value evaluation is reduced, even in gauged sites.

1 INTRODUCTION

The challenges faced in watershed modeling in recent years have reflected the need to deal with spatial variability and scaling and the need to consider explicitly linkage among hydrology, meteorology and climatology.

Some of the most important advances in watershed modeling have involved approaches that employ Geographic Information System (GIS). Most of the recent progresses reached in hydrologic field stimulated the revived interest in warning systems as a means to reduce negative consequences of floods: a viable alternative or an addition to traditional structural flood controls such as dams or levees. In fact a better and less expensive alternative to structural interventions, frequently referred as non-structural methods, is the development of more effective management methods for existing water resource system. Forecast can be an important tool in this framework.

Some natural environments are characterized by morphological structures having such a spatial dimension that the temporal gap between an intense rainfall event and the concerning flood is extremely short. In these particular environments, the forecasting tools operating in real time must refer to meteorological data, that remain in this context the only efficacious information about precipitation field. Due to the uncertainty associated to the predictions of storm development, the forecast must be performed for the whole event concerned on an extended geographic area. Hydrological models can be also used to create some scenarios. In this case the model is run on a synthetic precipitation data set. In this way, a set of probable basin response is created, on which excess frequency of flooding for certain areas can be estimated. Mapping flood-prone areas provides a fair warning of the risks of building in such places and should discourage unwise land use.

In the present work a rainfall–runoff model based on a territory description derived from data stored in Digital Elevation Models (DEMs) is presented. It has been developed in order to be a possible answer to flash flood forecasting in relatively small and steep basins. Due to the limited response time of such basins the model is able to use meteorological derived rainfall predictions as inputs. In this framework the major need is to find simple schematizations of the physical processes able to furnish a robust description capacity. This can be carried on even to the detriment of the accuracy in simulating the less interesting features of the hydrographs.

2 RIVER NETWORK EXTRACTION FROM DEMS

The first step to describe drainage morphology from DEMs' data is to choose an objective criterion to identify water pathways. The most common method is based on the concept of maximum slope: each pixel drains towards the once of its eight neighbors, along the direction of maximum slope; in this way the identification of the draining direction is unambiguous. By linking each pixel to the one immediately downstream, a space-filling representation of the drainage network is obtained. Even if this allows the immediate recognition of the entire catchment shape, through the identification of the watershed boundary, it does not represent a realistic description of the network. The main straits to obtain a more realistic representation of the drainage structure is in channel-head identification, since there is not an objective criterion to recognize the transition from overland flow to channeled flow, i.e. from hillslopes to the drainage network. Therefore, a filtering procedure based on a threshold concept which constraints hillslopes to assume the length required by overland flow to initiate surface erosion and, consequently, to form and maintain a channel must be introduced. Mongomery and Dietrich (1988) with a fields study, supported the hypothesis of a channelization threshold existence able to set a finite scale to the landscape: an empirically defined topographic threshold can be associated with the channel head location, which defines the border between undissected hillslopes and the valley bottom to which they drain.

The most common method of pruning the space-filling network consists in imposing a channelization area-threshold (Tarboton et al., 1992). This approach is equivalent to suppose that the channel growth is linked to water discharge, being the drained area proportional to the discharge itself (Leopold and Miller, 1956). The threshold value can be estimated from slope area plots; the presence of a consistent scatter in this plot limits this method. Furthermore, the identified network shows a constant drainage density all over the basin.

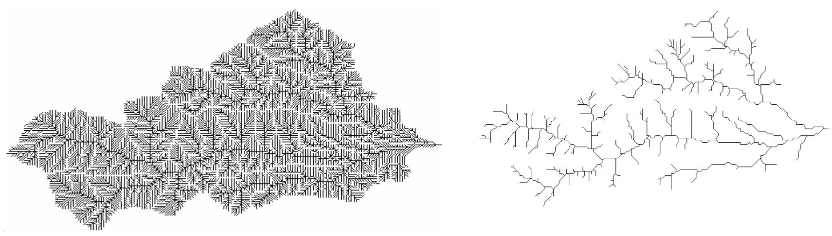

Figure 2.1: Space-filling (left) and filtered network (right), Centa river basin.

A more sophisticated approach is based on landscape curvature: the channel is a convergent element while the hillslope has a divergent topography, thus a single inflection point is expected in the longitudinal profile near the head of a channel (Smith and Bretherton, 1972), at which slope will achieve its maximum value. The applicability of this method is highly limited by the low resolution of DEM's data.

Assuming the basin morphology as the results of the interaction between soil properties and climate stress, erosion and transport processes play a fundamental role in shaping the territory, the shear stress, τ, is proportional to the local slope, S in the form: $\tau \propto SA^{0.5}$. Another criterion to identify the channel network can be based on the excess of a shear stress threshold. This means to consider a non-constant channel-head contributing area, in inversion proportion to the local slope: $A_S \propto 1/S$.

Recent studies agree in considering the constancy of the quantity AS^k for points belonging to the channel-network. This is based on the concept that the upper part of the basin, characterized by higher slope values, favors the erosion and the beginning of dissection. This approach supply a non-uniform drainage density in the different parts of the basin, accordingly with nature. Different theoretical models supply global relationships that consider drained area and local slope to identify the drainage network. In the following paragraphs the main approaches that support this filtering procedure will be summarized.

2.1 Global equation approach

The drainage network, as well as many other natural connecting patterns, is basically a transportation system for which the treelike structure is the most appealing from the point of view of efficiency in the construction, operation, and maintenance of the system. Rodriguez-Iturbe et al. (1992) postulated the three following energy principles, demonstrating they are sufficient to give rise a treelike drainage structure instead of different configuration:

- principle of minimum energy expenditure in any link of the network;
- principle of equal energy expenditure per unit area of channel anywhere in the network;
- principle of minimum energy expenditure in the network as a whole.

The first principle expresses a local optimal condition for any link of the network. The second expresses an optimal condition throughout the network regardless of its topological structure and makes all channels equally efficient when adjusted for size. It postulates that energy expenditure is the same everywhere in the network when normalized by the area of the channels on which takes place. The later is a global condition and it should be interpreted in a probabilistic framework. It is addressed to the topological structure of the network and refers to the optimal arrangement of its elements, considering the drainage system as a unique object.

Coupling the expressions obtained from the joint application of the two principles of energy expenditure, with constant velocity throughout the network it is obtained:

$$P_l = c_1 Q^{0.5} S + c_2 \tag{1}$$

For a given flow condition, c_1 and c_2 are constant throughout the network, and under the second principle P_l is constant in all links. Thus, applying the first two principles to the hydrodynamic mechanism able to shape the basin, results:

$$Q^{0.5} S = constant \tag{2}$$

Often, discharge measurements in every link of a network are not available and, since the mean annual flow has been observed to be proportional to the drainage area in many regions of the world, the contributing area may be used as a surrogate variable for discharge:

$$AS^k = constant \tag{3}$$

This relationship can be studied in detail using DEMs.

2.2 Hydrodynamic approach

A simplified hydrodynamic approach (Roth et al., 1996) that considers erosion and sediment transport processes through contributing area and slope as the fundamental authors in shaping catchment components can be developed. Under the hypothesis of uniform flow, the discharge can be evaluated by the Chezy equation.

As it has been observed by Leopold et al. (1964), a very good relationship exists between width, depth and the square root of discharge for both bank-full and mean annual flow. This corroborates the assumption of self-similarity of the cross section, hypothesis that leads the width and the hydraulic radius to be linearly dependent on the depth.

Dimensionless bottom shear stress all over the channel network can been expressed by the Shields equation.

On these basis it is possible to write:

$$QS^k \cong (\tau_* d_s)^f \tag{4}$$

that provides a link between the morphological variables, discharge and slope, and the hydrodynamics of the mechanism of sediment transport. Under the assumption of constant shear stress, equivalent to assume that the basin tends to reach a constant value of energy expenditure in the whole channel network, as proposed by Rodriguez-Iturbe et al. (1992), and using the area as a surrogate variable for discharge it is possible to obtain:

$$AS^k = constant \tag{5}$$

2.3 Morphologic approach

River systems present different fractal dimensions which are convenient descriptors of their scaling behavior. The fractal scaling can be observed at individual level, considering the single rivers wandering, or at global level considering the organization of river network structures. On the basis of self-similarity described by laws of stream length and stream areas (Horton, 1932), Rosso et al., 1991 reported that rivers are fractals with a fractal dimension *d.*

While on the basis of self-similarity described by laws of stream numbers and stream length, La Barbera and Rosso (1987) reported that the fractal dimension for river networks is given by *D*.

$$d = \max\left(1, 2\frac{\log R_l}{\log R_a}\right);\ D = \min\left[2, \max\left(1, \frac{\log R_b}{\log R_l}\right)\right] \tag{6}$$

where: R_l stream length ratio, R_a stream area ratio, R_b bifurcation ratio.

The scaling properties of the river network as a whole can be viewed as the product of the structural composition of the drainage system, reflected by *D* and the fractal nature of river length, described by *d*. By introducing this source of fractal behaviour of individual streams in eq (2.9). Tarboton et al. (1990) obtained the fractal measure $D_t = D \cdot d$.

Considering the same analysis, the properties of invariance and scaling in drainage network have been described by La Barbera and Roth (1994). They linked the fractal nature of river networks to the cumulative probability distribution of stream order, stream length, contributing area and energy dissipation. With regard to this consideration, a general cumulative probability distribution referred to the quantity AS^k has been derived:

$$P\left[\frac{AS^k}{A_\Omega S_\Omega^k} \geq \frac{A_* S_*^k}{A_\Omega S_\Omega^k}\right] = \frac{\left(\frac{R_l}{R_b}\right)^\Omega}{1-\left(\frac{R_l}{R_b}\right)^\Omega}\left(\left(\frac{A_* S_*^k}{A_\Omega S_\Omega^k}\right)^{-\frac{d}{2}(D-1)\frac{\Theta}{\Theta-k}} - 1\right) \tag{7}$$

This expression leads to $AS^k = constant$ for a critical exponent k_c.

$$\Theta = k_c = -\frac{\log R_a}{\log R_s} \tag{8}$$

where: Ω maximum stream order within the basin, R_s stream slope ratio.

A good agreement between the predicted and the observed distribution was found, with the exception of the low values of contributing areas. Since this expression refers only to the channel path, any deviation, can be assumed to be an indicator of the presence of hillslope points. Therefore the comparison between observed cumulative frequency distribution and theory leads to identify cells that do not belong to the channel network for which this distribution (AS^k *= constant)* has been derived.

2.4 Filter application

A good schematization of the drainage network is fundamental in hydrology, and many automated network identification procedures starting from DEM's give as final result a space-filling description. For hydrologists, interested in describing the hydrological response of the drainage systems, the plane filling tree obtainable directly from DEM's is not a suitable representation of the network, since the transport processes on the hillslopes are quite different from that of the drainage network. Thus it is necessary to set a physical limit to the upstream development of the channeled network. This kind of approach is not new to the fractal description of natural objects: it is expected that there will exist upper and lower limits to the scaling behavior depending on the physical characteristics of the processes involved. In this case the physical limit is the hillslope length, which is linked with channel initiation and maintenance processes.

All the three methods described above drive to the conclusion that there is a strong correlation between the morphologic variables contributing area and local slope and the real drainage structure through the equation $AS^k = constant$. A filter which makes use of this equation, is expected to give a good estimation of the real drainage network, in fact this geomorphologic approach, takes into account a lot of aspects which contribute to shape the drainage network structure of the basin.

This filtering procedure, based on the concept that in the channel network the quantity AS^k is greater than a particular value, is used as a threshold criterion that discriminates the different transport mechanisms in channeled and not channeled paths. In some previous works (Roth et al., 1996) this filter was applied in order to draw the river network, using a target value for drainage density as a means to guaranty physic control to the procedure. The authors in fact, enforced the drainage density to assume a pre-determined value, substantially in agreement with natural observations of the network. We apply the filtering procedure without imposing any limit to the bifurcation growth, or any external condition to be respected. We performed the comparison between the threshold value and the local value of the AS^k quantity. The procedure is applied following the scheme below:

1. pit removal;
2. evaluation of a quasi-local slope in substitution of the local slope, taking into account the upper and downstream links of the cell studied, this to avoid many problems connected to the presence of sampling errors in DEMs;
3. computation of the quantity AS^k in each pixel of the basin grid;
4. application of the slope-area filter: pixels characterized by threshold value greater than the threshold chosen represent the effective drainage structure for that particular filter area. Scanning the matrix from mountain to valley zones, when a channel-pixel is found, the channel is traced to the outlet

following the pointer path, eliminating unrealistic channel path interruptions.

The river network derived in this way, well reproduces the natural networks, showing a non-uniform drainage density all over the basin: in the mountain part the high value of slope promotes the channel initiation.

In applying this procedure, the first problem faced has been the evaluation of the threshold value; in fact, in literature there were no previous works that applied this filtering approach in the rainfall-runoff modeling context. Here, the estimation of the threshold value was performed with reference to different targets. A first graphic control is that the river network identified must be reasonable and close to the cartography's blue lines, but others theoretically based controls are possible. The theory on which these type of works are based, states that the value of the quantity AS^k is constant in the channeled network. Thus, theoretically, the percentage of channel would reach asymptotically a quasi-stable value by increasing the threshold and then drop dramatically due to the river network vanishing when this value is exceeded. This behavior is not strictly respected in nature; however, it is possible to observe an asymptotic trend in the neighborhoods of the threshold value. The basin morphology produced by the filtering procedure distributes the hillslope's lengths within the basin. The hillslope's length frequency distribution, obtained by using particular threshold values, is heterogeneous accordingly with nature and, as suggested by Gyasi-Agyei et al. (1996) it is well described by a Gamma function with its exponential tail. While this takes into account the presence of hillslope, it is also important the spatial distribution of their lengths. All the processes that contribute to the channel initiation, as erosion and sediment transport processes, depend on the local morphology of the territory, than a non random spatial hillslope lengths distribution is expected. With regard to these consideration 100000 m^2 has been chosen as threshold value.

Great importance has the other morphologic parameter, the k exponent of the threshold expression, that is the responsible of drainage density redistribution within the basin. This parameter assigns different importance to the slope: it magnifies the importance of high value of slope, in the steepest zones, while it dignifies importance to the slope in the flat areas. Several papers in literature deal with the problem of k estimation. Studies by Flint (1974) suggested a range between *1.2* and *2.7*. In this work, $k=1.7$ has been chosen as a good value, that represents also the average of the interval proposed by Flint.

3 MODEL STRUCTURE

A rainfall - runoff model right to embrace problems concerning orography and urbanization characteristics of the Liguria region is developed.

The model is simple and is addressed to the description of some characteristics of the hydrograph; it does not describe the infiltration processes in detail, solving the continuity equation by using an empirical method. It is a calibrated parameter model: its parameters have been evaluated by fitting computed hydrographs to observed ones, although it is possible to control the parameters values, giving them an intuitive and direct physical meaning. It is called semi-distributed model, because it takes into account the spatial variation of inputs such as rainfalls, morphologic, geological and anthropic characteristics of the basin, but it is almost lumped in parameters and outputs even if results can be obtained wherever in the catchment. It is intended as a general model in a wide class of relatively small watersheds (from 10 to 1000 km^2).

The first part of the model's procedure provides to distinguish hillslopes and channel network paths by using the geomorphologic filter described in section 2. The basin routing time is evaluated assigning different typical velocity values in each pixel pertaining to the basin and classified as hillslope or channel. Summing the time that water spends respectively on the hillslope and in the channel, a routing time is obtained for each pixel as:

$$\tau_i = \frac{L_{hi}}{v_h} + \frac{L_{ci}}{v_c} \tag{9}$$

The two velocities, assumed constant for each of the two components of the drainage system maintain their physical implication related to a particular class of extreme events and therefore to a particular range of discharge.

The basin response is determined using the Instantaneous Unit Hydrograph technique. Actually, there are some differences between the general way of proceeding and the model: here each pixel that contributes to generate the response of the basin is considered separately from the others, as regards the runoff generation at any temporal step. This allows the representation of the dispersion runoff generation due to both rainfall and soil characteristic heterogeneity, adding a dispersion degree that lets the model to better approach the real behavior of the basin. The routine is therefore strictly related to the way of interpolating rainfall depth data and the local rainfall runoff transformation, which makes use of the S.C.S method with the Curve Number parameters locally defined, according to soil type and soil cover.

3.1 Parameters

The validity of the model resides in its simplicity both in inputs and in the limited number of parameters that have to be calibrated resulting parsimonious and robust. The model makes use of five parameters. Two are addressed to describe the geomorphology of the environment where the model is applied. They are the threshold value AS^k and the geomorphologic exponent k. Both of them have a long literature background with regard to morphologic studies (Roth et al., 1996), but they were never investigated within a rainfall-runoff modeling context. The physically based calibration was successfully performed on these parameters as explained in detail in § 2.4. Two other parameters link the geomorphic structure of the basin with the hydrologic processes that take place on the different components of the drainage network. They are two characteristic velocities one for the hillslope links and the other for the channel paths. In this way the scale of the Geomorphologic Instantaneous Unit Hydrograph (GIUH), considered the weak point of this kind of approach (Shalmseldin and Nash, 1998), is defined within a physically based framework directly linked to pluviometric and hydrometric records. The calibration of these parameters were performed on the basis of several historical observations in different basins through a procedure of calibration and further validation. The direct physical meaning of these two parameters – they can be interpreted as the average velocity on hillslope and channel links – allows us to control the values coming out of the calibration procedure. The last parameter is not to be interpreted as a real parameter of the model, but as a degree of freedom of the model itself. It describes the moisture condition antecedent to the time the model runs. It is obviously an event sensitive parameter and has to be introduced by the operator any time a simulation is performed.

4 HISTORICAL EVENTS SIMULATIONS

A large number of intense rainfall events in different Ligurian basins of various sizes, ranging from 100 to 1000 km^2, have been collected and used to calibrate the parameters. A best-fit was performed on the peak and time to peak. The numerical results of the simulations are by far satisfying if looking at the main hydrograph characteristics. The global shape of the hydrograph is not very well preserved and this was to be expected by the construction of the model itself. The model in fact does not manage the infiltrated part of rainfall, which has a relatively unimportant role in the characterization of the hydrograph in the peak neighborhoods at least in the environments analyzed.

A unique set of parameters, regardless of the event analyzed and the basin considered, leads to interesting results. This is by far the most important characteristic of the model for its utilization both in flood forecasting and in the regionalization procedure as proposed in the present work.

As desirable, in a geomorphologic and climatic homogeneous region as Liguria, the two parameters describing the morphologic characteristics of the environment remain constant all over the region. On the other hand it was not possible to foresee such a behavior also for the kinematic parameters. Moreover the velocities' values, $v_{hillslope} = 0.16\ ms^{-1}$ and $v_{channel} = 2.5\ ms^{-1}$, obtained are realistic and this contributes to enforce the idea that the model allows a good representation of the hydrographic response at a basin scale when forced by intense rainfall events along the northern Mediterranean coast. The parameters invariance has some major implications which are not to be renounced in using this model as a supporting tool in the regionalization study. The invariance with regard to rainfall intensity is important because the calibration of the parameters is usually carried out on a limited set of data, which is not able to cover all the range of rainfall intensities concerned with the simulations: the model performs in conditions sometimes clearly different from the ones encountered during the calibration of the parameters. The invariance with regard to basin size allows the model's utilization in non gauged basins. The whole Ligurian environment is constituted by a multitude of small non gauged basins which often represents the major source of problems within the region. This point is of foremost importance in the regionalization procedure which is addressed to widen the knowledge on gauged basins to a broad class of catchments in an homogeneous region. In other words it would not be possible to apply in this procedure any other calibrated model but one supplying this characteristic.

5 RESULTS AND CONCLUSIONS

The DRiFt model is used here as a crucial unit for the discharge regionalization procedure in the Liguria region. The validation of the procedure was carried out considering DRiFt as a tool inserted in a more sophisticated modus operandi. This chain contemplates as first step a work which deals with the rainfall regionalization in the same homogeneous region (Boni, this volume). Many critic decisions were pointed out during the application of the regional flood frequency analysis (e.g. shape and return period evaluation for the design ietograph, Curve Number regional re-calibration) the reader is asked to refer to Ferraris et al., (this volume) for a detailed explanation of the validation procedure which has led to the following results and comments.

In Figure 5.1 and Figure 5.2 dimensionless regional growth curve, together with its confidence limit with 0.05 significance level, is proposed as final result of the global study presented in Ferraris et al., (this volume). In Figure 5.1 the curve is compared with the historical data made dimensionless with the single site time-series: it is possible to observe the good agreement between the two. Figure 5.2 shows the comparison between the growth curve and some simulated discharges in gauging sites uniformly distributed all over the region. They are made dimensionless with the simulated discharge derived from a T=2.9 years design ietograph, which shall give the expected discharge value for each single site. In this second case the agreement is less satisfactory.

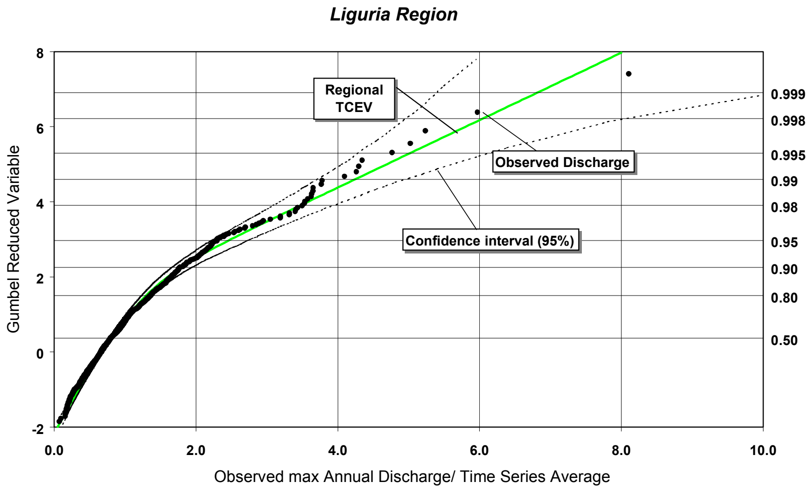

Figure 5.1: Comparison between dimensionless historical data and growth curve.

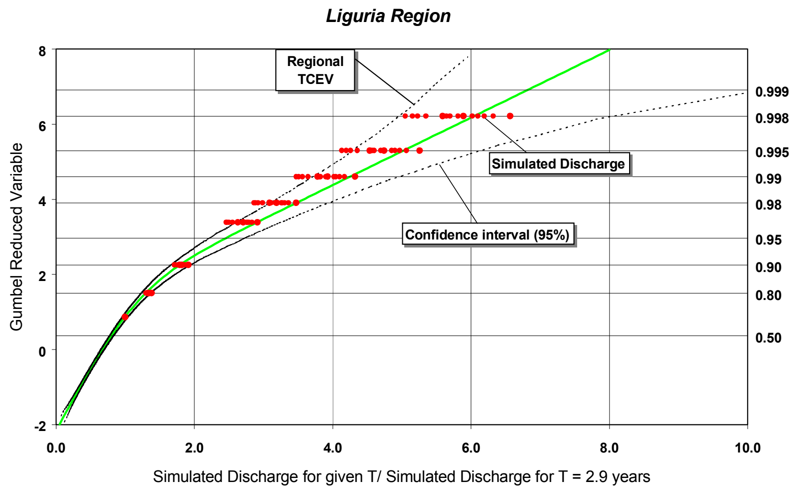

Figure 5.2: Comparison between dimensionless DRiFt simulations and growth curve.

Actually some comments are necessary. As explained by Arnell and Gabriele (1984) the TCEV distribution is extremely sensitive to outliers capturing. The presence of data which show frequencies so low that they are not possible to evaluate on the basis of the sample size, affects the parameters estimates. Sometimes this lead to misleading results. In our opinion this is the case of the single data circled in Figure 5.3. An improvement in the accordance between the growth curve and both data and simulations can be obtained by recomputing the TCEV parameters excluding this single value. Even if this has to be better justified by statistical tests and empirical observations based on a better knowledge of this particular event, results shown in Figure 5.3 and Figure 5.4 are quite promising.

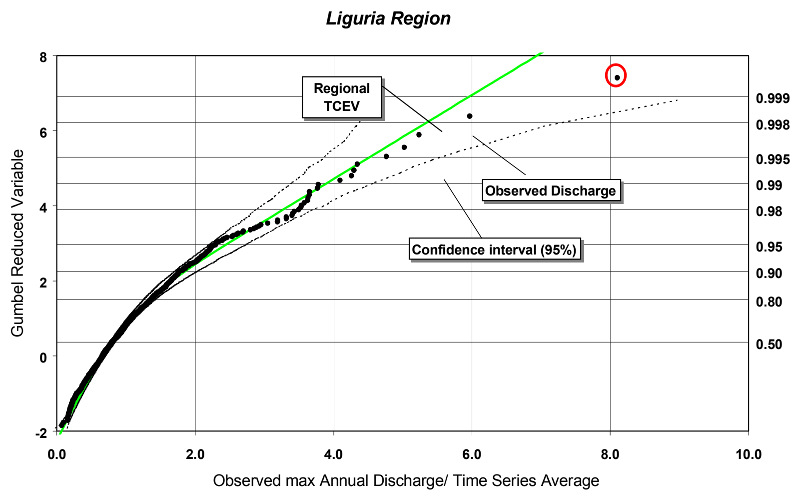

Figure 5.3: Comparison between dimensionless historical data and growth curve (TCEV parameters calculated without the circled point).

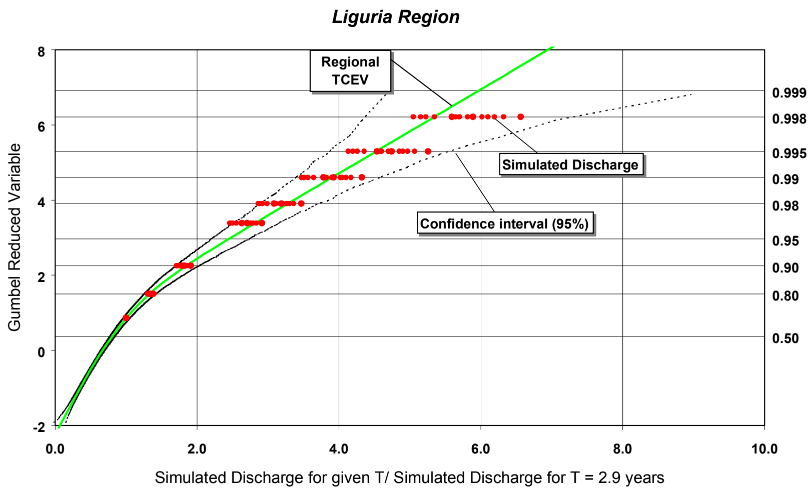

Figure 5.4: Comparison between dimensionless DRiFt simulations and growth curve (TCEV parameters calculated without the circled point).

6 REFERENCES

Arnell, N.W. and Gabriele, S. The performance of the two component extreme value distribution in regional flood frequency analysis, *Water Resour. Res.*, 1984, **20**, 879-887.

Boni, G., A physically based regional rainfall frequency analysis: application to a coastal region in northern Italy, this volume.

Ferraris, L., Parodi, U. and Roth G. Regional flood frequency analysis through a rainfall-runoff model: application to a coastal region in northern Italy, this volume.

Flint, J.J. Stream gradient as a function of order, magnitude and discharge. *Water Resour. Res.,* 1974, **10**(5), 969-973.

Gyasi-Agyei, Y., F. P. de Troch, and P. A. Troch, A dynamic hillslope response model in a geomorphology based rainfall-runoff model, *J. Hydrol.,* 1996, **178**, 1-18.

Horton, R.E. Drainage basin characteristic, *Trans. Am. Geophys. Union*, 1932, **13**, 113-123.

La Barbera, P. and Rosso, R. Fractal geometry of river networks, *Eos Trans. Am. Geophys. Union*, 1987, **68**, 1276

La Barbera, P. and Roth, G. Invariance and scaling properties in the distributions of contributing area and energy in drainage basins, *Hydrol. Proc. J.*, 1994, **8**, 125-135.

Leopold, L. B. and Miller, J. P. Ephemeral streams-Hydraulic factors and their relation to the drainage network, *U. S. Geol. Surv.*, 1956, Prof. Pap. 282-A.

Rodriguez-Iturbe, I., Rinaldo, A., Rigon, R., Bras, R. L., Marani, A. and Ijjasz-Vasquez, E. Energy Dissipation, Runoff Production, and the Three-Dimensional Structure of River Basins, *Water Resour. Res.*, 1992, **28**(4), 1095-1103.

Rosso, R., Bacchi, B. and La Barbera, P. Fractal Relation of Mainstream Length to Catchment Area in River Networks, *Water Resour. Res.*, 1991, **27**(3), 381-387.

Roth, G., La Barbera, P. and Greco, M. On the description of the basin drainage structure, *J. Hydrol.,* 1996, **187**, 119-135,.

Shamdeldin, A. Y. And Nash, J. E. the geomorphological unit hydrograph – a critical review, *Hess,* 1998, **2**(1)*,* 1-8

Smith, T. R. and Bretherton, F. P. Stability and the conservation of mass in drainage basin evolution, *Water Resour. Res.,* 1972, **8**(6), 1506-1529.

Tarboton, D. G., Bras, R. L. and Rodriguez-Iturbe, I. The fractal nature of the river networks, *Water Resour. Res*., 1988, **24**(8), 1317-1322.

Tarboton, D. G., Bras, R. L. and Rodriguez-Iturbe, I. Comment on: On the fractal dimension of stream networks, by La Barbera, P. and Rosso, R., *Water Resour. Res.*, 1990, **26**, 2243-2244.

Tarboton, D. G., Bras, R. L. and Rodriguez-Iturbe, I. A physical base for drainage density, *Geomorphology*, 1992, **5**(1-2), 59-76.